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INSOLATION IN RELATION TO CLOUD AMOUNT

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ABSTRACT

Assuming a simple pyrheliometric station model, an equation is derived relating the amount of insolation Q from a sky whose fraction C is covered by clouds, to the insolation Q_0 arriving at the same surface from a cloudless sky. The equation is of the form

$$Q/Q_0 = 1 - (A - a)(1 + a)C,$$

where A is the sum of cloud albedo and absorptivity expressed as fraction of the radiation incident on cloud tops and the symbol a represents the depletion coefficient of insolation in cloudless air in the layer between cloud top and cloud base levels.

The theoretical equation resembles the empirical equation $Q/Q_0 = 1 - kC$ where k is supposed to be a constant. The theoretical equation shows the dependence of k on relevant physical variables.

It is shown that the theoretical equation combined with results of pyrheliometric observations, from which a value of k has been deduced, leads to a value of A which is in close accord with its value obtained by independent methods. On the other hand, if we assume reasonable values for A and a, the resulting value for k is in good agreement with the best value found from pyrheliometric observations.

INTRODUCTION

In the literature on empirical calculations of insolation, an equation of the type

$$Q = Q_0 (1 - kC) \qquad (1)$$

is often cited as a useful means for computing, in an approximate manner, the amount of insolation reaching a horizontal surface from a cloudy sky under average conditions. In the equation the symbols have the following meaning:

$Q=$ amount of insolation incident on a horizontal surface from a cloudy sky,

$Q_0=$ as Q but from a cloudless sky,

$C=$ cloud amount on the scale of, say, 0.0 to 1.0,

$k=$ a suitable constant.

For simplicity, k is considered a constant in spite of the fact that k is known to vary with cloud type and depth,

height of cloud base, etc. It would be desirable to have quantitative information on the dependence of k on the various cloud characteristics and the purpose of the present paper is to investigate the problem in a simple manner. Such an investigation is all the more timely as the "best" values reported for k differ in some cases fairly considerably.

DERIVATION OF AN EQUATION

We will assume a simplified station model. In this model: (a) all clouds have their bases and tops at respectively uniform levels, (b) the optical properties, i. e., albedo and absorptivity, of clouds are uniform with respect to insolation, (c) transmission conditions of insolation in the cloud-free spaces of a partly cloudy sky are similar to those of the corresponding layer under conditions of a cloudless sky, and (d) the station is situated in a level terrain whose surface is uniform from an optical point of

view. At any level in the atmosphere, the insolation has a downward component and an upward component. The upward component is produced by reflections of insolation from the air, clouds, and terrain. In the present paper, the downward component alone is considered.

Let Q_0'' and Q' be the insolation incident on a horizontal surface at the cloud-top and cloud-base levels, respectively. For a sky partly covered by clouds, Q' should be thought of as a *weighted average* for the cloudy and cloudless areas on the constant-height surface corresponding to cloud-base level. Let $(1-a)$ be the transmission coefficient (a is the depletion coefficient) of insolation *in cloudless air* between the levels of cloud tops and cloud bases. Further, let A be the sum of figures representing cloud albedo and absorptivity expressed as fractions of the radiation incident on the cloud tops. On the one hand, for the cloud-free spaces, the insolation arriving at cloud base level will be proportional to $Q_0''(1-a)$. On the other hand, the insolation arriving at the bases of clouds will be proportional to $Q_0''(1-A)$. Hence Q', the weighted areal average of insolation reaching cloud-base level, will be

$$Q'=Q_0''(1-a)(1-C)+Q_0''(1-A)C,$$
$$=Q_0''[(1-a)-(A-a)C], \quad (2)$$

where, as before, C is the cloud amount expressed as fraction of the sky covered by clouds. For a cloudless sky, $C=0$ and $a=0$, by definition.

Let a' and a'' be transmission coefficients of insolation: a' for air between cloud-base level and ground, and a'' for *cloudless* air between cloud-top level and ground. Then, if Q and Q_0 have the same meaning as in (1),

$$Q=a'Q' \quad (3)$$

and

$$Q_0=a''Q_0'' \quad (4)$$

The latter is based on our assumption (c) according to which transmission conditions in the cloudless spaces of a partly cloudy sky are similar to those of a cloudless sky. Inserting (3) and (4) in (2) yields

$$Q=Q_0 \frac{a'}{a''}[(1-a)-(A-a)C]. \quad (5)$$

As a'' is the transmission coefficient in cloudless air between cloud-top level and ground whereas a' is the transmission coefficient for the layer between cloud-base level and ground, a'' will be smaller than a' by a factor representing the transmission coefficient for the layer of cloudless air between cloud-top and cloud-base levels. The latter coefficient has been denoted earlier by the symbol $(1-a)$. Hence,

$$\frac{a'}{a''}=\frac{1}{1-a} \quad (6)$$

If (6) is inserted in (5), we find that

$$Q=Q_0\left[1-\frac{A-a}{1-a}C\right] \quad (7)$$

If cloud depth is not too great, $a \ll 1$, and then, to a good approximation,

$$\frac{1}{1-a}=1+a \quad (8)$$

With the aid of (8), (7) now takes the form

$$Q=Q_0[1-(A-a)(1+a)C] \quad (9)$$

This is the equation we have intended to derive.

DISCUSSION

Equation (9) is the general form of the relationship between Q, Q_0, and C under the simplified model adopted. This equation shows the dependence of the coefficient of cloudiness (which in the empirical equation (1) is considered a constant) on the appropriate physical variables such as cloud albedo and absorptivity and transmission conditions in cloudless air for insolation. As both A and a are subject to temporal and spatial variations, a loss of accuracy will be incurred if the coefficient $(A-a)(1+a)$ of (9) is replaced by a constant as is done in (1). Equation (9) enables us to assess the error in replacing $(A-a)(1+a)$ by a constant provided, of course, that A and a can be estimated in a satisfactory manner.

Equation (6) shows that a is a function of the transmission coefficients a' and a''. For a given solar zenith distance, the transmission coefficients tend to vary with altitude as the logarithm of altitude. Klein [1] has reviewed this subject and compiled two diagrams (his figures 2 and 3, p. 125), one for summer conditions and the second for winter conditions, indicating the variation of transmission coefficients with altitude as a function of solar zenith distance, *for direct solar radiation*. In the absence of more complete data, we shall be compelled in the next section, where a numerical application is given, to use Klein's figures as if they were applicable for total insolation, direct and diffuse. The influence of this error is reduced by the fact that the value of the factor $(A-a)(1+a)$ is determined primarily by the value of A. This follows from the observation that an error in the value of a is compensated through a appearing with opposite signs in the two factors $(A-a)$ and $(1+a)$. This fact is of some usefulness in applications of equation (9) as it will often be difficult to estimate a to a sufficient accuracy.

APPLICATION: CLOUD ALBEDO AND ABSORPTIVITY FOR SOLAR RADIATION

Two immediate applications of equation (9) suggest themselves. In the first application, the factor $(A-a)$

$(1+a)$ is considered a constant and it is assumed that its "best" value is known. By assuming also that a is known, we can proceed to calculate the value of A, the sum of fractions representing cloud albedo and absorptivity for solar radiation, and compare the value of A so obtained with values derived by independent methods. The second application is the converse of the first one. By assuming that representative values of A and a are given, the value of the cloudiness coefficient $(A-a)(1+a)$ can be computed and then checked for reasonable agreement with the value of k (equation (1)) determined empirically on the basis of pyrheliometric observations.

First application.—In equation (9) it will be assumed that $(A-a)(1+a)=k$, k=constant, in which case (9) reduces to (1). Some writers (e. g., Sverdrup [2], p. 51), give the "best" value of k as 0.71. Dr. S. Fritz of the U. S. Weather Bureau has, however, pointed out, in a private communication, that if average monthly periods are considered, the value of k is more likely near 0.6 or even near 0.5. These estimates of k are based on pyrheliometric observations. On the basis of this, we shall adopt tentatively $k=0.55$.

With regard to a, we note from equation (6) that

$$a=\frac{a'-a''}{a'} \qquad (10)$$

To obtain a value for a, we shall estimate a' and a'' from Klein's ([1], p. 125) diagrams. By assuming a cloud base at 1, 2, and 3 km., and a cloud depth of 1, 2, and 3 km., it is found that a varies between ca. 0.1 and 0.2. We shall adopt for a the average of the two figures, i. e. we assume $a=0.15$. As was pointed out in the preceding section, errors in estimates of a are partly balanced by the manner in which the a enters the cloudiness coefficient.

From $(A-a)(1+a)=0.55$ and $a=0.15$, $A=0.63$, that is, the sum of cloud albedo and absorptivity works out to be 0.63. This value is in close agreement with values obtained recently by independent methods. Both Fritz (private communication) and Houghton [3] give the average value of cloud albedo as 0.55. Fritz's estimate was presented in 1951 at the Brussels General Assembly of the International Union of Geodesy and Geophysics. The estimate falls in the interval 0.47 and 0.60 given earlier by Fritz [4] for the worldwide average for cloud albedo. As to cloud absorptivity, Houghton [3] presents a table, based on the work of various authors, indicating an absorptivity of 0.01 for high clouds, at the one extreme, and 0.1 for cumuliform clouds, at the other extreme. It is probably a reasonable conjecture that the worldwide average is nearer the value for cumuliform clouds, say, it is 0.07. If the latter is combined with the figure 0.55 for albedo, it results that $A=0.62$, in close agreement with the value 0.63 deduced above.

Second application.—As was stated earlier, the second application is the converse of the first one. Setting out from a cloud albedo of 0.55, an absorptivity of, say, 0.07 and a value of $a=0.15$, equation (9) becomes

$$Q=Q_0(1-0.54\ C)$$

i. e., k of (1) is 0.54, in good accord with Fritz's estimated value of k based on pyrheliometric observations.

ACKNOWLEDGMENT

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